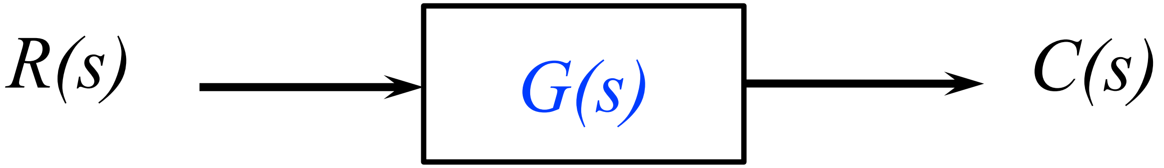
GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

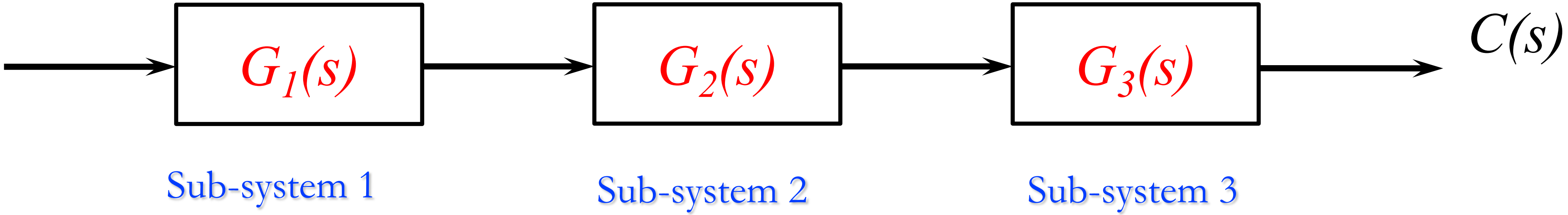
Graphical Models

Block Diagram

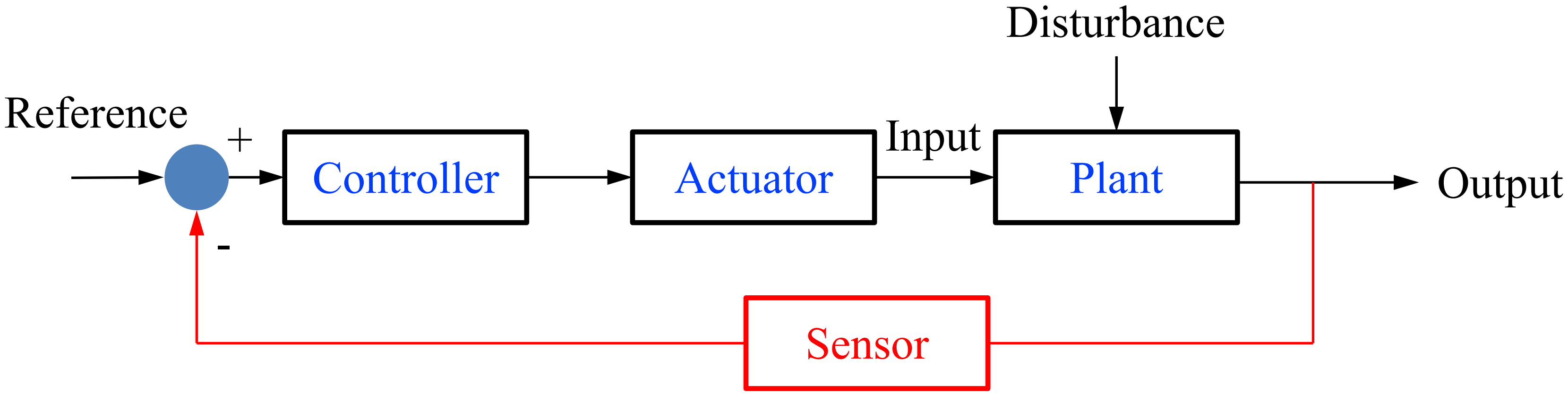
Block Diagram



Systems usually are composed of multiple subsystems:

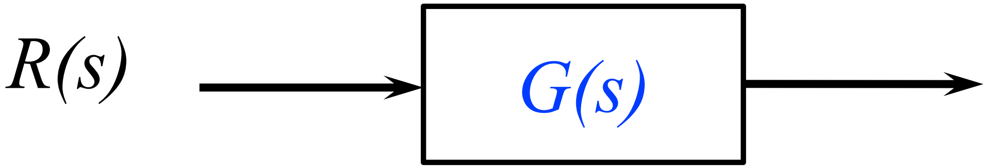


More complex control block diagram (e.g., Feedback)

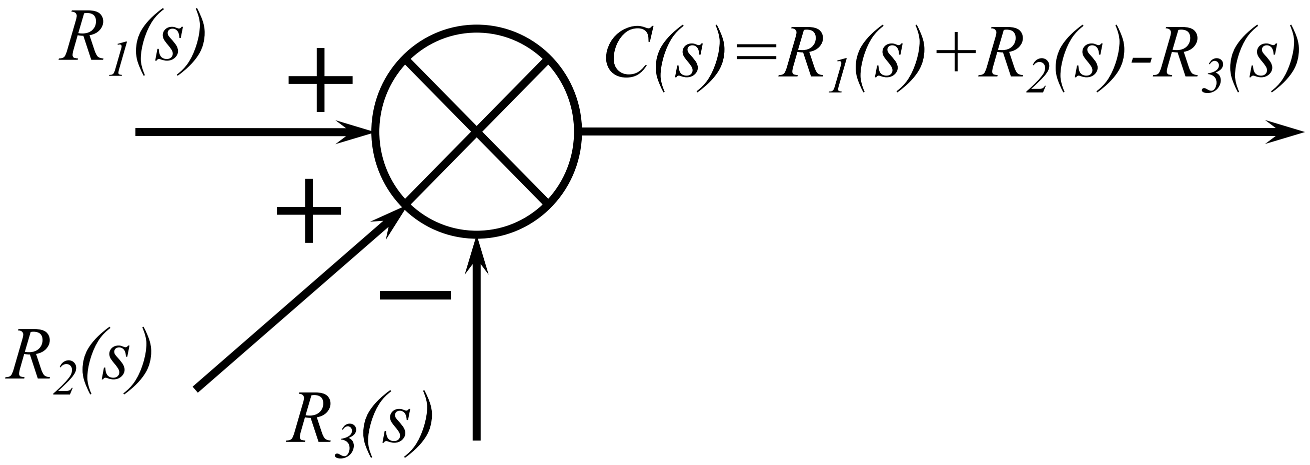


Basic Components of Block Diagrams

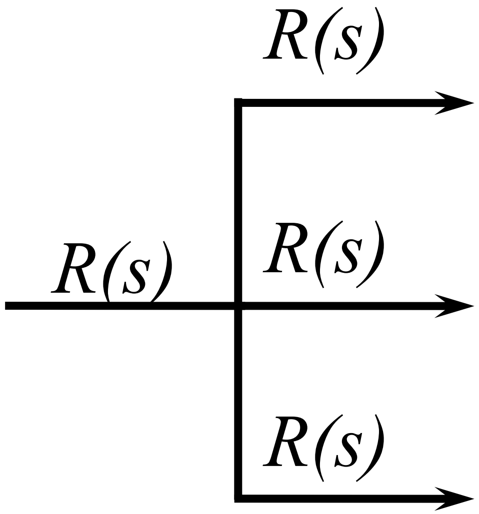
Signals



System

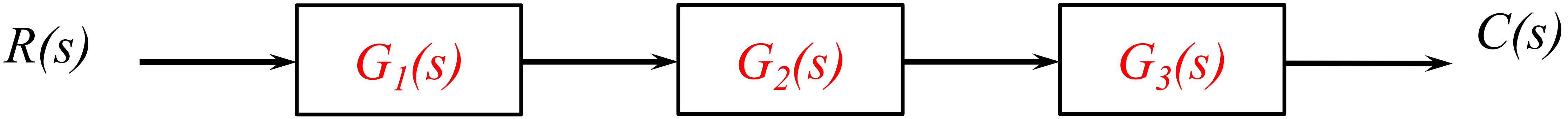


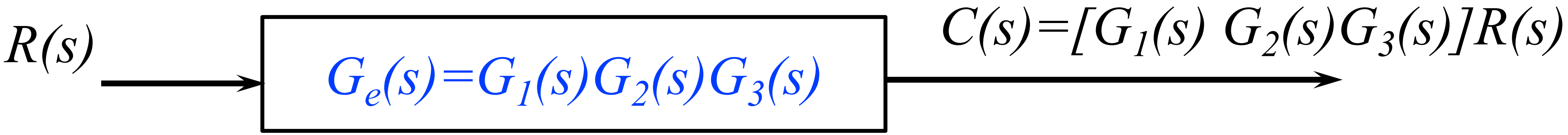
Summing Junction



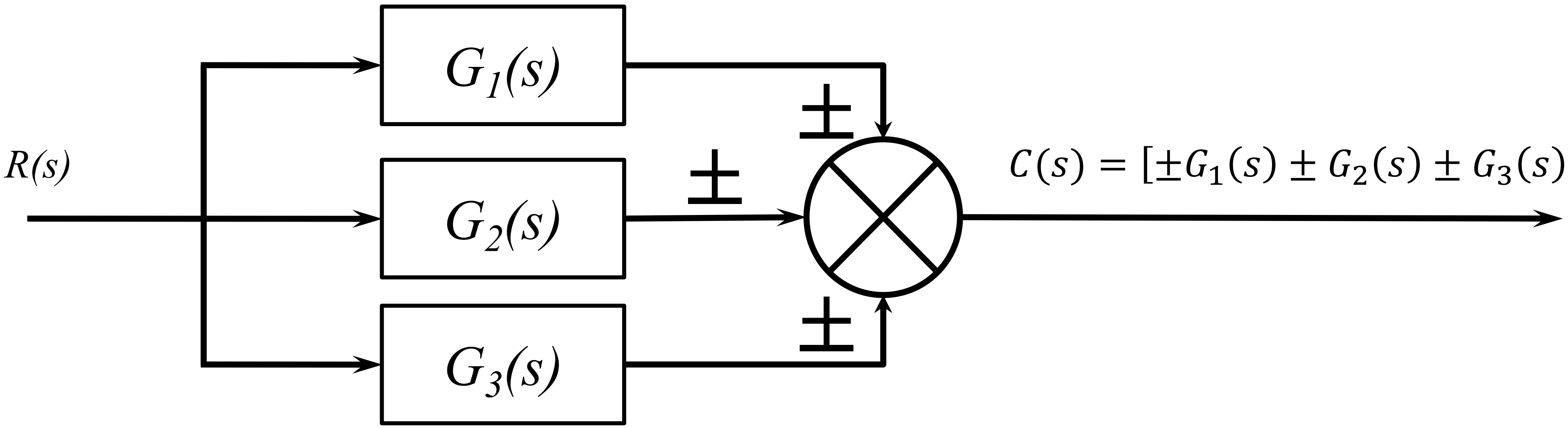
Pickoff Point

Cascade Form

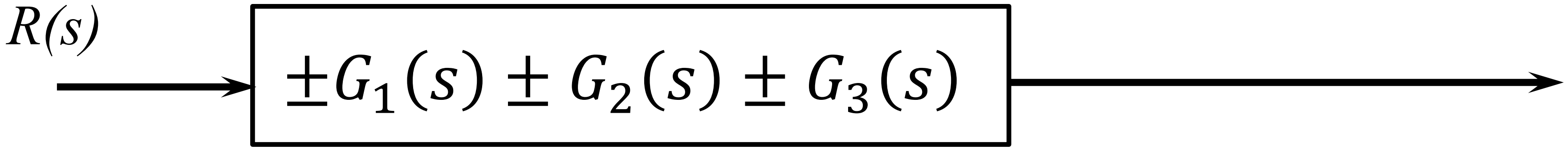




Parallel Form

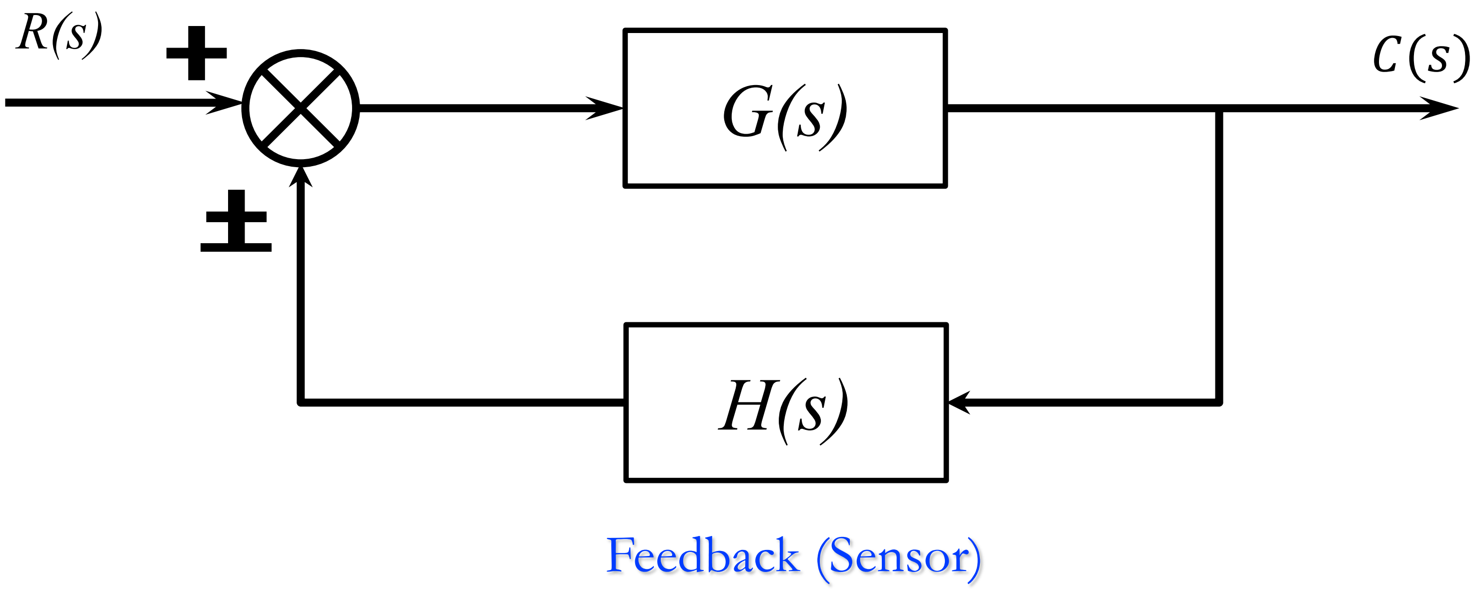


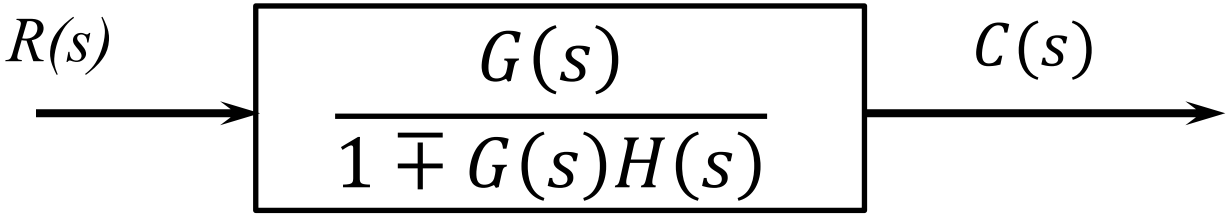
$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$



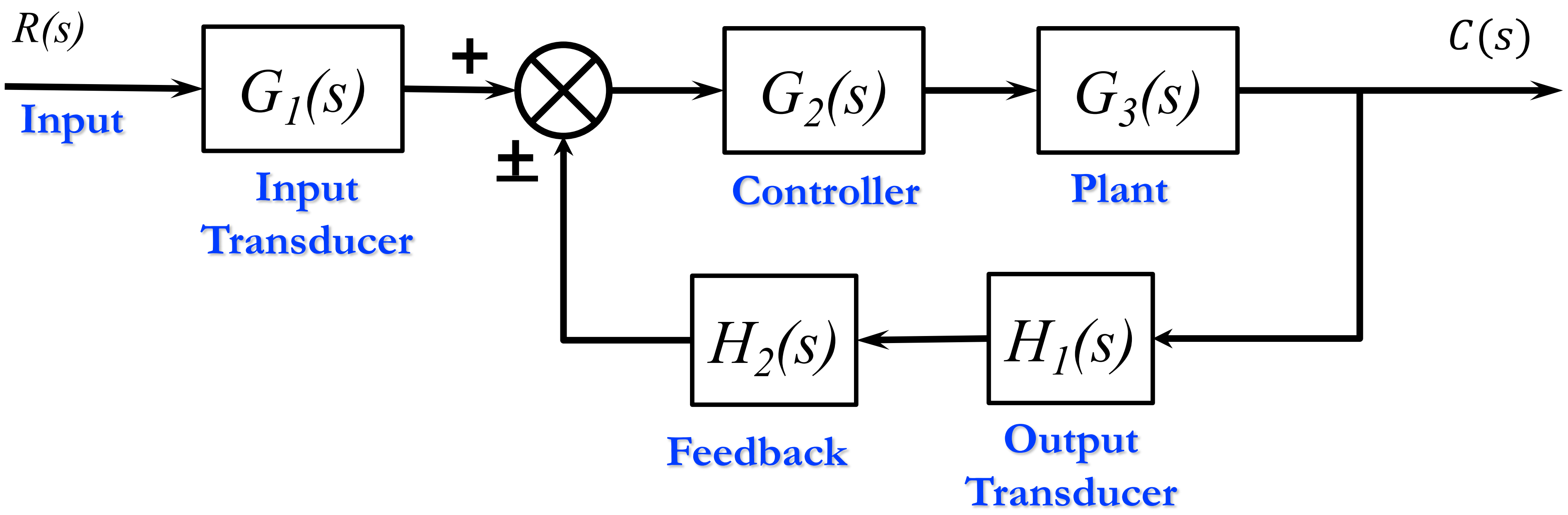
$\pm G_1(s) \pm G_2(s) \pm G_3(s)$

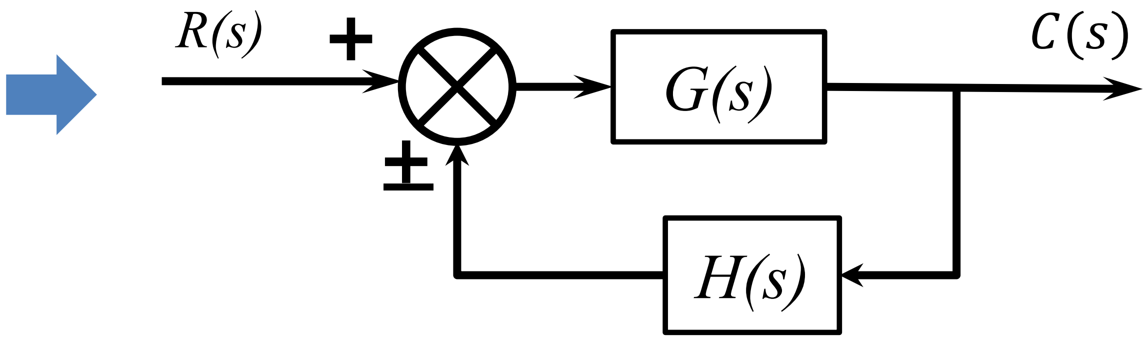
Feedback Form: Eliminating a Feedback Loop

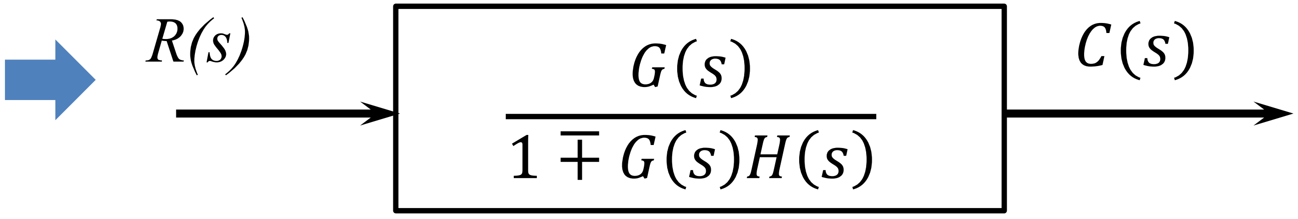




Moving Blocks to Create Familiar Forms

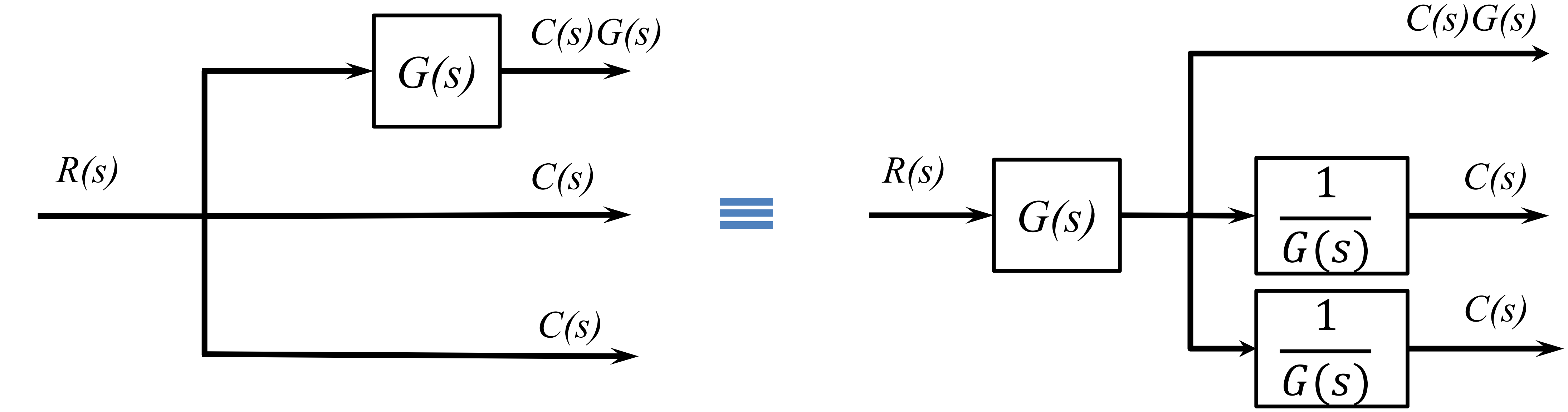


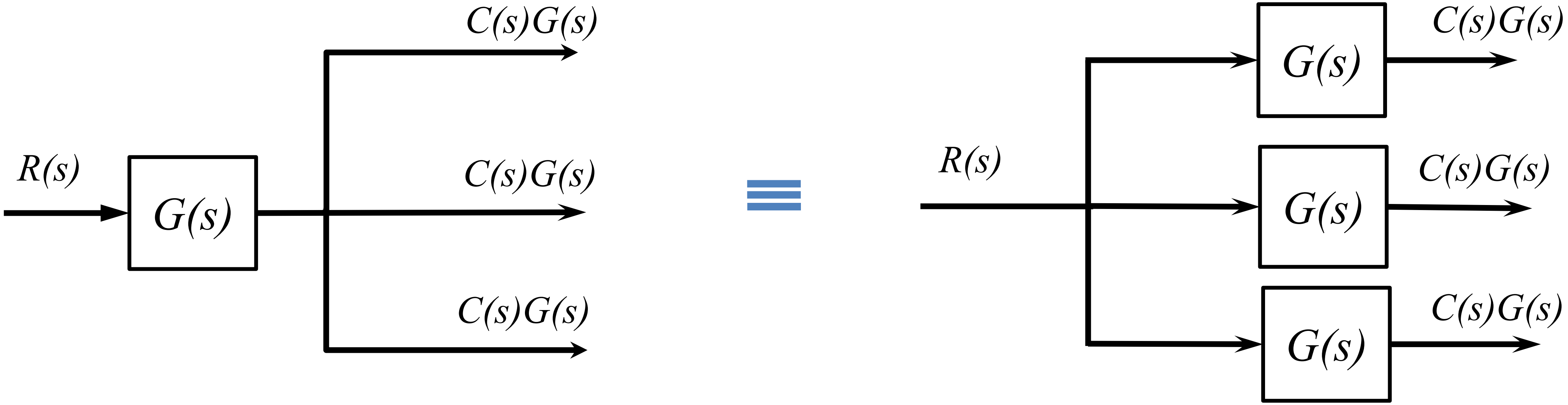




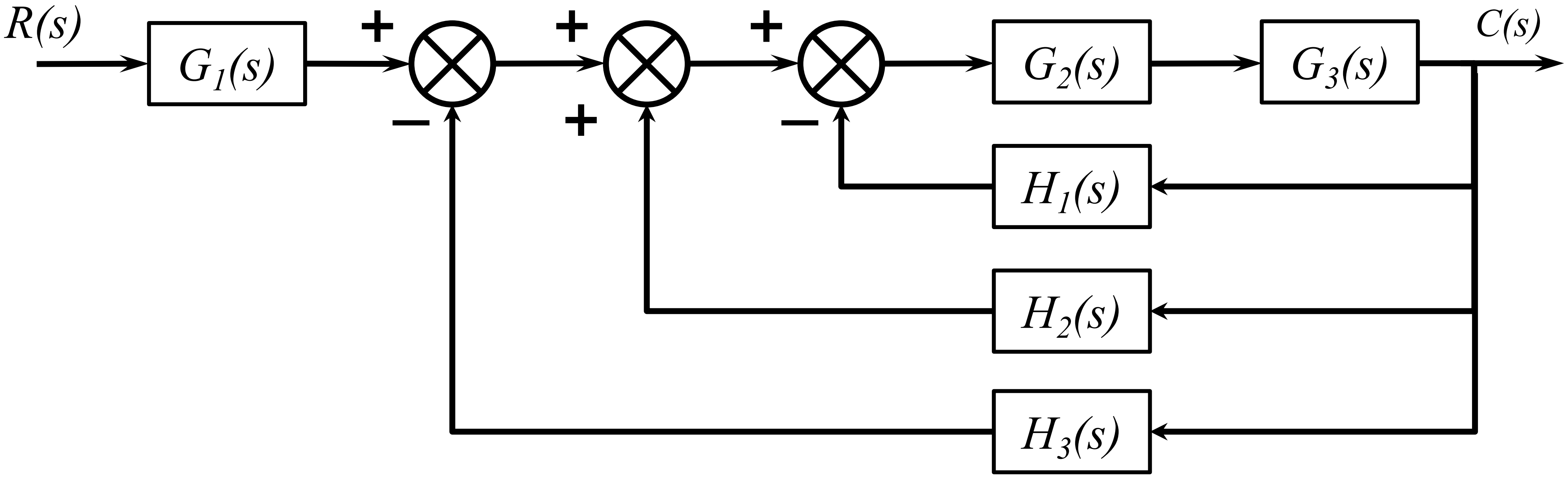
Moving a Summing Junction

Moving a Pickoff Point



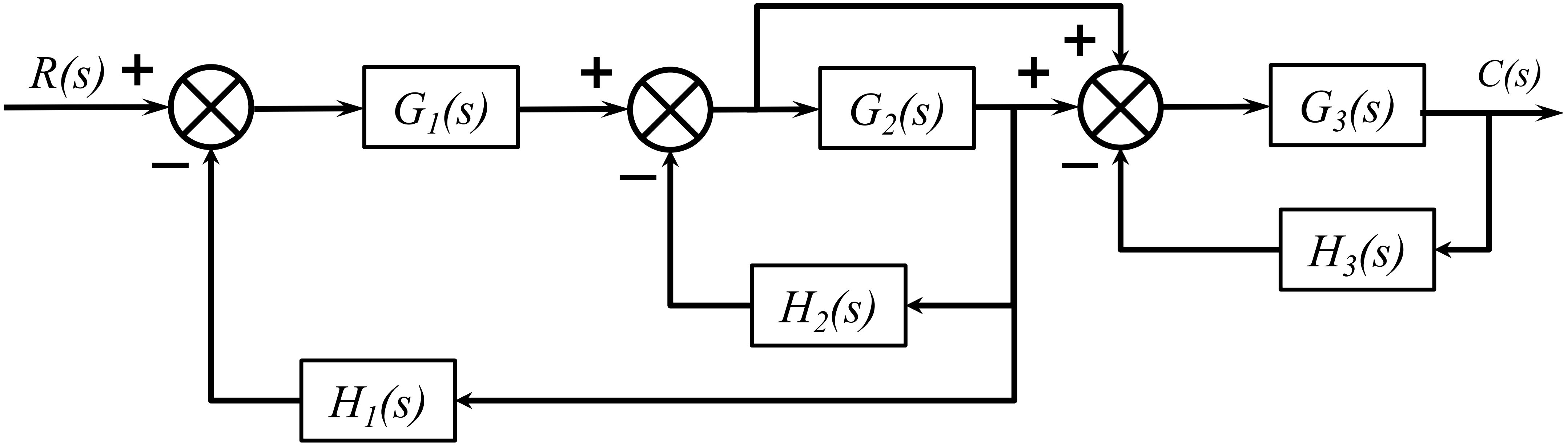


Example 1



Example 1 (Cont'd)

Example 2

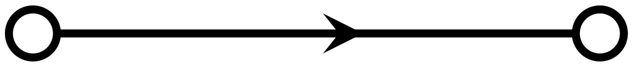


Example 2 (Cont'd)

Signal Flow Graph

Definition

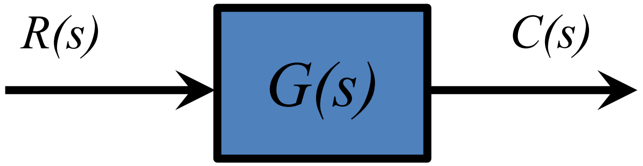
A system is represented by a line with an arrow showing the direction of signal flow through the system.

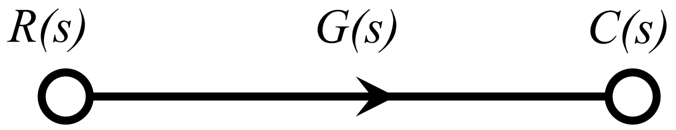


A signal-glow graph consists only **branches** and **nodes**:

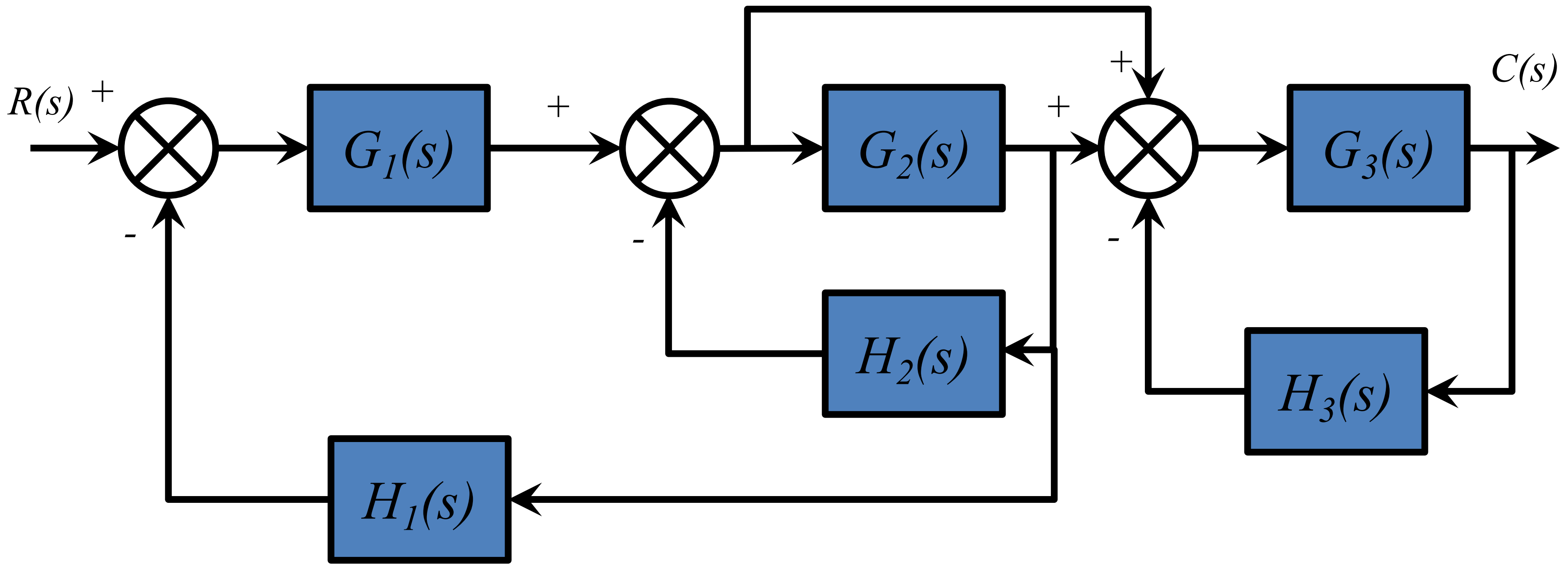
Branches: represent **systems**

Nodes: represent **signals**





Example



Mason's Rule: Definition

Loop Gain:

The product of branch gains found by traversing a path that starts at a node and ends at the same node, following the direction of the signal flow, without passing through any other node more than once.

Forward-path Gain:

The product of gains found by traversing a path from the input node to the output node of the signal-flow graph in the direction of signal flow.

Non-touching Loops:

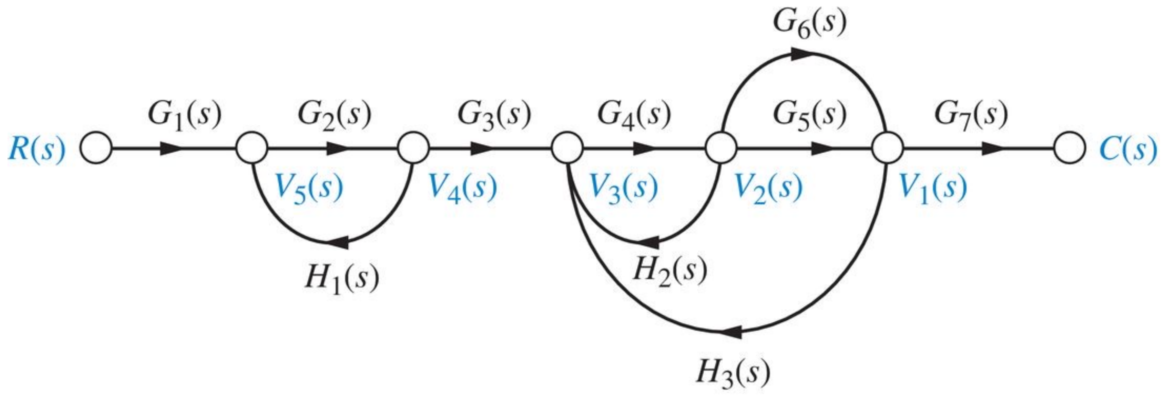
Loops that do not have any nodes in common.

Non-Touching-Loop Gain:

The product of loop gains from non-touching loops taken two, three four, or more at a time

Example

Loop Gain:



Forward-path Gain:

Non-touching Loops:

Non-Touching-Loop Gain:

Mason's Rule

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

k = number of forward paths

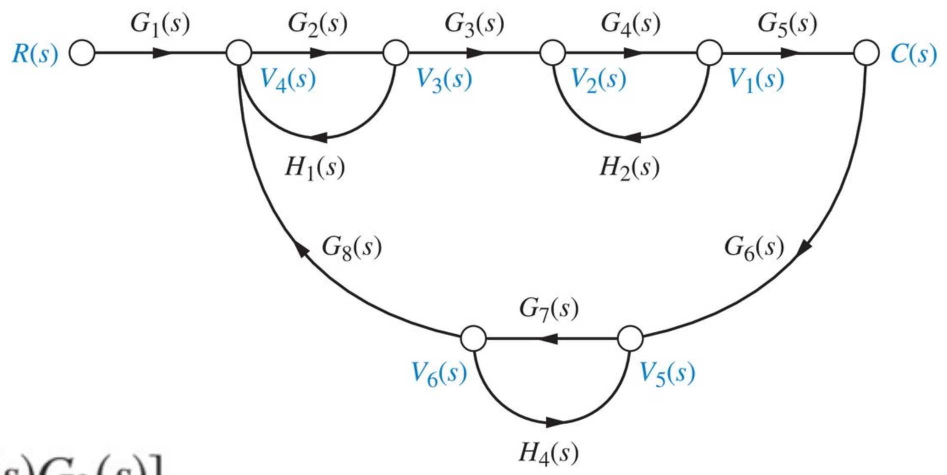
T_k = the *k*th forward-path gain

Δ = 1- Σ loop gains + Σ non-touching loop gains taken two at a time – Σ non-touching loop grains taken three at a time + Σ non-touching loop gains taken four at a time …

Δ_k = Δ – Σ loop gain terms in Δ that touch the *k*th forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the *k*th forward path.

Example

Find the transfer function, $C(s)/R(s)$ for the signal-flow-graph:



$$G(s) = \frac{T_1\Delta_1}{\Delta} = \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta}$$

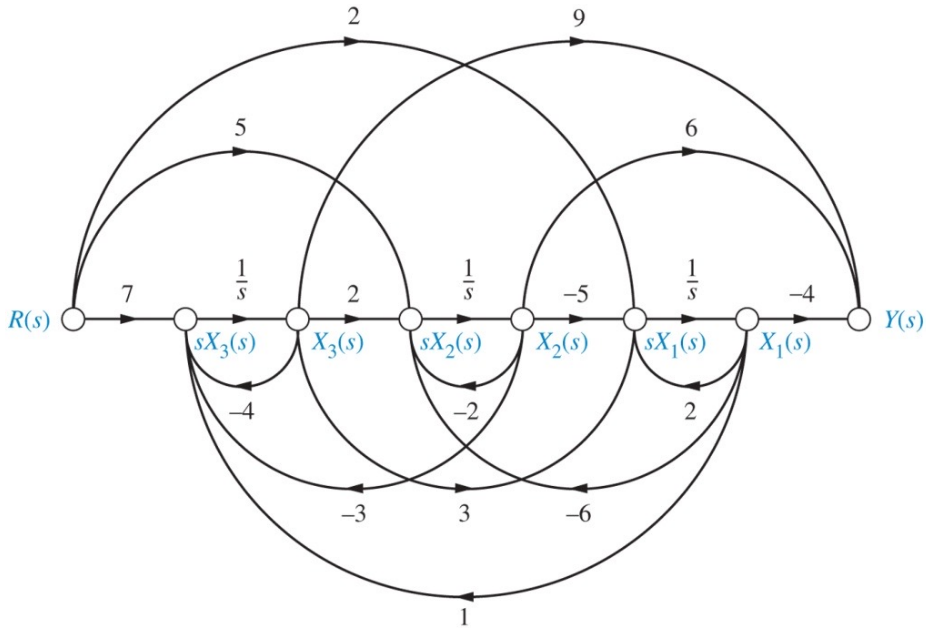
$$\begin{aligned}\Delta = 1 &- [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s) \\ &\qquad + G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)] \\ &+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s) \\ &\qquad + G_4(s)H_2(s)G_7(s)H_4(s)] \\ &- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]\end{aligned}$$

Signal-Flow Graphs of State Equations

Consider the following state and output equations:

$$\begin{cases} \dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r \\ \dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r \\ \dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r \\ y = -4x_1 + 6x_2 + 9x_3 \end{cases}$$

where r is the input, y is the output, x_1, x_2 and x_3 are the state variables, please draw its signal-flow graph.



The End!!